Math 236, additional problems for Homework #4

These problems are due, along with the rest of Homework #4, at the beginning of class on Monday, January 23.

A1. Let F_n denote the Fibonacci sequence ($F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$). Prove that for all $n \geq 1$,

$$F_1 + \cdots + F_n = F_{n+2} - 1.$$

A2. Prove that for all $n \geq 1$,

$$F_1^2 + \cdots + F_n^2 = F_n F_{n+1}.$$

A3. Formulate a conjecture about the value of the expression $F_n^2 + F_{n+1}^2$ (for $n = 1, 2, 3, \ldots$) in terms of other Fibonacci numbers. Then prove your conjecture is correct.

A4. Let the "Tribonacci sequence" be defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for all $n \geq 4$. Prove that $T_n < 2^n$ for all $n \geq 1$.